|  |  |
| --- | --- |
| Division | 10th |
| Subject | Mathematics |
| Chapter | Number system |
| Author | Ruhani Kashni |
| Category | 03 |

|  |
| --- |
| The square of any positive integer cannot be of the form  (2018) |
|  |
|  |
|  |
|  |
| C |
| we consider the remainder when n2 is divided by 5  If n leaves a remainder of 1 when divided by 5, then n2 will leave a remainder of 1 when divided by 5.  If n leaves a remainder of 2 when divided by 5, then n2 will leave a remainder of 4 when divided by 5.  If n leaves a remainder of 3 when divided by 5, then n2 will leave a remainder of 4 when divided by 5.  If n leaves a remainder of 4 when divided by 5, then n2 will leave a remainder of 1 when divided by 5. |
| Let be the positive integer and .  Then, by Euclid's algorithm, some integer and because a .  or or or or  .  , where is a positive integer.  , where is a non-negative integer.  , where is a non-negative integer.  , where is a non-negative integer.  , where is a non-negative integer.  The square of any positive integer is of the for  i.e., it cannot be of the form for a integer |
| Real numbers introduction |

|  |
| --- |
| There is a circular path around a sports field. Priya takes 18 minutes to drive one round of the field. Harish takes 12 minutes. Suppose they both start at the same point and at the same time and go in the same direction. After how many minutes will they meet?  (2019) |
| 36 minutes |
| 18 minutes |
| 6 minutes |
| They will not meet |
| A |
| using the least common multiple (LCM) of their individual times to complete one round of the field.  Priya's time to complete one round = 18 minutes.  Harish's time to complete one round = 12 minutes.  Find the LCM of 18 and 12: To find the LCM, you can list the multiples of each number and find the smallest number that appears in both lists.  Multiples of 18: 18, 36, 54, 72, 90, Multiples of 12: 12, 24, 36, 48, 60, 72,. |
| The time it takes for them to meet again can be found using the concept of the least common multiple (LCM) of their individual times to complete one round of the field.  Priya's time = 18 minutes Harish's time = 12 minutes  The LCM of 18 and 12 is 36, which means they will meet again after 36 minutes |
| Real numbers introduction |

|  |
| --- |
| Three farmers have 490 kg, 588 kg and 882 kg of wheat respectively. Find the maximum capacity of a bag so that the wheat can be packed in exact number of bags.  (2019) |
| 98 kg |
| 290 kg |
| 200 kg |
| 350 kg |
| A |
| Step 1: List the amounts of wheat each farmer has  Farmer 1: 490 kg  Farmer 2: 588 kg  Farmer 3: 882 kg  Step 2: Find the common factors of the given amounts. The GCD is the largest number that divides all the given amounts without leaving a remainder.  Factors of 490: 1, 2, 5, 7, 10, 14, 35, 49, 70, 98, 245, 490  Factors of 588: 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84, 98, 196, 294, 588  Factors of 882: 1, 2, 3, 6, 7, 9, 14, 18, 21, 42, 49, 63, 98, 126, 147, 294, 441, 882  Step 3: Identify the common factors |
| The amounts of wheat each farmer has are:  Farmer 1: 490 kg  Farmer 2: 588 kg  Farmer 3: 882 kg  Now, let's calculate the GCD of these amounts:  GCD (490, 588, 882) = 98 |
| Decimal expansions of real numbers |

|  |
| --- |
| The LCM of two numbers is 1200. Which of the following cannot be their HCF?  (2018) |
| 600 |
| 500 |
| 400 |
| 200 |
| B |
| We know that LCM of two or more numbers is always divisible by their HCF. 1200 is divisible by 600, 200 and 400 but not by 500 .  To find the HCF of two numbers, we need to consider the factors they have in common. Let's consider some pairs of numbers that multiply to 1200:  a=600 and b=2 (since 600⋅2=1200  a=300 and b=4 (since 300⋅4=1200  a=400 and b=3 (since 400⋅3=1200 |
| Given that the LCM of two numbers is 1200,  we know that: a⋅b=1200.  H=500, then we need to find two numbers a and b such that  a⋅b=1200 and the HCF of a and b is 500.  the HCF of all these pairs is either 2, 4, or 1 (since 1 is the smallest positive integer that divides any number). This means that there are no two numbers whose HCF is 500 and whose product is 1200. |
| Decimal expansions of real numbers |

|  |
| --- |
| On a morning walk, three persons step off together and their steps measure 40 cm, 42 cm, and 45 cm, respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps?  (2017) |
| 2520cm |
| 2525cm |
| 2555cm |
| 2828cm |
| A |
| Given step lengths:  Person 1: 40 cm  Person 2: 42 cm  Person 3: 45 cm  To find the LCM, we can follow these steps:  Step 1: Prime factorization of the step lengths:  40 = 23 × 5  42 = 2 × 3 × 7  45 = 32 × 5  Step 2: Identify the highest powers of all prime factors:  Step 3: Calculate the LCM |
| We need to find the L.C.M of 40, 42 and 45 cm to get the required minimum distance.  40 = 2×2×2×5  42 = 2×3×7  45 = 3×3×5  L.C.M. = 2×3×5×2×2×3×7 = 2520 |
| Euclid's Division Lemma |

|  |
| --- |
| **For any positive integer n, n3 – n is divisible by**  **(2015)** |
| 4 |
| 5 |
| 6 |
| 7 |
| c |
| Let n be any positive integer. And since any positive integer can be of form 6q, or 6q+1, 6q+2, 6q+3, 6q+4, or 6q+5. (From Euclid’s division lemma for b= 6) |
| We have **n3 – n = n(n2-1) = (n-1) n(n+1),**  For n= 6q,  ⇒ (n-1) n(n+1) = (6q-1) (6q) (6q+1)  ⇒ (n-1) n(n+1) = 6[(6q-1) q(6q+1)]  ⇒ (n-1) n(n+1) = 6m, which is divisible by 6. [m= (6q-1) q(6q+1)]  For n= 6q+1,  ⇒ (n-1) n(n+1) = (6q) (6q+1) (6q+2)  ⇒ (n-1) n(n+1) = 6[q(6q+1) (6q+2)]  ⇒ (n-1) n(n+1) = 6m, which is divisible by 6. [m= q(6q+1) (6q+2)]  For n= 6q+2,  ⇒ (n-1) n(n+1) = (6q+1) (6q+2) (6q+3)  ⇒ (n-1) n(n+1) = 6[(6q+1) (3q+1) (2q+1)]  ⇒ (n-1) n(n+1) = 6m, which is divisible by 6. [m= (6q+1) |
| Euclid's division lemma |

|  |
| --- |
| **The product of two consecutive positive integers is divisible by**  **(2011)** |
| 2 |
| 3 |
| 4 |
| 5 |
| a |
| Let’s consider two consecutive positive integers as (n-1) and n.  ∴ Their product = (n-1) n  = n2 – n  We know that any positive integer is of form 2q or 2q+1. (From Euclid’s division lemma for b= 2) |
| consider two consecutive positive integers as (n-1) and n.  ∴ Their product = (n-1) n  = n2 – n  We know that any positive integer is of form 2q or 2q+1. (From Euclid’s division lemma for b= 2)  So, when n= 2q  We have,  ⇒ n2 – n = (2q)2 – 2q  ⇒ n2 – n = 4q2 -2q  ⇒ n2 – n = 2(2q2 -q)  Thus, n2 – n is divisible by 2.  Now, when n= 2q+1  We have,  ⇒ n2 – n = (2q+1)2 – (2q-1)  ⇒ n2 – n = (4q2+4q+1 – 2q+1)  ⇒ n2 – n = (4q2+2q+2)  ⇒ n2 – n = 2(2q2+q+1)  Thus, n2 – n is divisible by 2 |
| Fundamental theorem of arithmetic |

|  |
| --- |
| **Find the HCF of 135 and 225**  (2016) |
| 25 |
| 35 |
| 45 |
| 55 |
| c |
| The integers given here are 225 and 135. On comparing, we find 225 > 135.  So, by applying Euclid’s division lemma to 225 and 135, we get  225 = 135 × 1 + 90 |
| The integers given here are 225 and 135. On comparing, we find 225 > 135.  So, by applying Euclid’s division lemma to 225 and 135, we get  225 = 135 × 1 + 90  Since the remainder ≠ 0, we apply the division lemma to the divisor 135 and remainder 90.  ⇒ 135 = 90 × 1 + 45  Now, we apply the division lemma to the new divisor 90 and the remainder 45.  ⇒ 90 = 45× 2 + 0  Since the remainder at this stage is 0, the divisor will be the HCF.  Hence, the HCF of 225 and 135 is 45. |
| Fundamental theorem of arithmetic |

|  |
| --- |
| **Find the HCF of the pairs of numbers:32 and 54**  (2019) |
| 1 |
| 2 |
| 3 |
| 4 |
| B |
| 54 = 32 × 1 + 22  Since remainder ≠ 0, apply division lemma on 32 and remainder 22  32 = 22 × 1 + 10 |
| Apply Euclid’s division lemma on 54 and 32  54 = 32× 1 + 22  Since remainder ≠ 0, apply division lemma on 32 and remainder 22  32 = 22 × 1 + 10  Since remainder ≠ 0, apply division lemma on 22 and remainder 10  22 = 10 × 2 + 2  Since remainder ≠ 0, apply division lemma on 10 and remainder 2  10 = 2 × 5 + 0  Therefore, the HCF of 32 and 54 is 2. |
| Relation between LCM and HCF of two numbers |

|  |
| --- |
| **Find the HCF of the pairs of numbers :70 and 30**  (2020) |
| 10 |
| 20 |
| 30 |
| 40 |
| A |
| Step 1 **Divide 70 by 30:**  70=30×2+1070=30×2+10  **Step 2 Now, divide 30 by 10:**  30=10×330=10×3 |
| 70 = 30 × 2 + 10.  Since remainder ≠ 0, apply division lemma on divisor 30 and remainder 10  30 = 10 × 3 + 0.  Therefore, the HCF of 70 and 30 is 10. |
| Relation between LCM and HCF of two numbers |

|  |
| --- |
| **Find the largest number, which divides 615 and 963, leaving the remainder 6 in each case.**  (2017) |
| 27 |
| 37 |
| 57 |
| 87 |
| d |
| The numbers are 615 – 6 = 609 and 963 – 6 = 957.  Now, if the HCF of 609 and 957 is found, that will be the required number |
| The numbers are 615 – 6 = 609 and 963 – 6 = 957.  Now, if the HCF of 609 and 957 is found, that will be the required number.  By applying Euclid’s division lemma  957 = 609 × 1+ 348  609 = 348 × 1 + 261  348 = 261 × 1 + 87  261 = 87× 3 + 0.  ⇒ HCF = 87. |
| Decimal representation of rational numbers in terms of terminating |

|  |
| --- |
| **If the HCF of 408 and 1032 is expressible in the form 1032m – 408** × **5, find m.**  (2016) |
| 1 |
| 2 |
| 3 |
| 4 |
| b |
| HCF of 408 and 1032 is to be found.  By applying Euclid’s division lemma, we get  1032 = 408× 2 + 216.  Here, the remainder ≠ 0. So apply Euclid’s division lemma on divisor 408 and remainder 216  408 = 216 × 1 + 192. |
| HCF of 408 and 1032 is to be found.  By applying Euclid’s division lemma, we get  1032 = 408× 2 + 216.  Here, the remainder ≠ 0. So apply Euclid’s division lemma on divisor 408 and remainder 216  408 = 216 × 1 + 192.  As the remainder ≠ 0, again apply the division lemma on divisor 216 and remainder 192  216 = 192 × 1 + 24.  Again, the remainder ≠ 0. So, apply the division lemma again on divisor 192 and remainder 24  192 = 24 × 8 + 0.  Now, it is seen that the remainder is 0.  Hence, the last divisor is the HCF of 408 and 1032, i.e., 24  So, this HCF is expressed as a linear combination, that is,  24 = 1032m – 408 × 5  1032m = 24 + 408× 5  1032m = 24 + 2040  1032m = 2064  m = 2064/1032  ∴ m = 2 |
| Decimal representation of rational numbers in terms of terminating |

|  |
| --- |
| **If the HCF of 657 and 963 is expressible in form 657x + 963 x – 15, find x.**  (2014) |
| 21 |
| 22 |
| 23 |
| 24 |
| b |
| HCF of 657 and 963 is to be found.  By applying Euclid’s division lemma, we get  963 = 657 x 1+ 306. |
| HCF of 657 and 963 is to be found.  By applying Euclid’s division lemma, we get  963 = 657 x 1+ 306.  Here, the remainder ≠ 0, so we apply Euclid’s division lemma on divisor 657 and remainder 306  657 = 306 x 2 + 45.  Now, continue applying the division lemma till the remainder becomes 0.  306 = 45 x 6 + 36.  Again, the remainder ≠ 0  45 = 36 x 1 + 9.  Again, the remainder ≠ 0  36 = 9 x 4 + 0.  Now, the remainder = 0.  Hence, the last divisor is the HCF of 657 and 963, i.e., 9  So, this HCF is expressed as a linear combination which is given as,  9 = 657x + 963 (-15).  Solving for x, we get  9 = 657x —14445  9 + 14445 = 657x  14454 = 657x  ⇒ x = 14454 / 657  ∴ x = 22 |
| Decimal representation of rational numbers in terms of non-terminating recurring decimals |

|  |
| --- |
| **An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?**  (2014) |
| 2 |
| 6 |
| 7 |
| 8 |
| d |
| The maximum number of columns = HCF of 616 and 32.  By applying Euclid’s division lemma  616 = 32 x 19 + 8 |
| It is given that an army contingent of 616 members is to march behind an army band of 32 members in a parade. Also, the two groups are to march in the same number of columns.  Thus, we need to find the maximum number of columns in which they can march.  This is done by simply finding the HCF of the given two numbers.  Therefore, the maximum number of columns = HCF of 616 and 32.  By applying Euclid’s division lemma  616 = 32 x 19 + 8  32 = 8 x 4 + 0.  So, HCF = 8 |
| Decimal representation of rational numbers in terms of non-terminating recurring decimals |

|  |
| --- |
| **A merchant has 120 litres of oil of one kind, 180 litres of another and 240 litres of the third kind. He wants to sell the oil by filling the three kinds of oil in tins of equal capacity. What should be the greatest capacity of such a tin?**  (2018) |
| 60 |
| 50 |
| 40 |
| 30 |
| a |
| The greatest capacity of the tin for filling three different types of oil can be found by simply finding the HCF of the three quantities 120, 180 and 240.  By Euclid’s division lemma on 180 and 120.  180 = 120 x 1 + 60 |
| The greatest capacity of the tin for filling three different types of oil can be found by simply finding the HCF of the three quantities 120, 180 and 240.  By Euclid’s division lemma on 180 and 120.  180 = 120 x 1 + 60  120 = 60 x 2 + 0 (here, the remainder becomes zero in this step)  Since the divisor at the last step is 60, the HCF (120, 180) = 60.  Now, let’s find the HCF of 60 and the third quantity of 240.  Applying Euclid’s division lemma, we get  240 = 60 x 4 + 0  And here, since the remainder is 0, the HCF (240, 60) is 60. |
| Revisiting irrational numbers |

|  |
| --- |
| **During a sale, colour pencils were being sold in packs of 24 each and crayons in packs of 32 each. If you want full packs of both and the same number of pencils and crayons, how many of each would you need to buy?**  (2013) |
| 1 |
| 2 |
| 3 |
| 4 |
| c |
| Number of crayons in a pack = 32.  So, the least number of both colour pencils and crayons that need to be purchased is given by their LCM. |
| Number of colour pencils in a pack = 24  Number of crayons in a pack = 32.  So, the least number of both colour pencils and crayons that need to be purchased is given by their LCM.  L.C.M of 24 and 32 = 2 x 2 x 2 x 2 x 2 x 3 = 96  ∴ The number of packs of pencils to be bought = 96 / 24 = 4 packs.  the number of packs of crayons to be bought = 96 / 32 = 3 packs. |
| Revisiting irrational numbers |

|  |
| --- |
| **144 cartons of Coke Cans and 90 cartons of Pepsi Cans are to be stacked in a Canteen. If each stack is of the same height and is to contain cartons of the same drink, what would be the greatest number of cartons each stack would have?**  (2011) |
| 16 |
| 17 |
| 18 |
| 19 |
| c |
| Number of cartons of coke cans = 144  Number of cartons of Pepsi cans = 90.  By Euclid’s division lemma on 144 and 90, we get  144 = 90 x 1 + 54 |
| Number of cartons of coke cans = 144  Number of cartons of Pepsi cans = 90.  So, the greatest number of cartons in a stack can be found by finding the HCF(144, 90).  Thus, by applying Euclid’s division lemma on 144 and 90, we get  144 = 90 x 1 + 54  90 = 54 x 1+ 36  54 = 36 x 1 + 18  36 = 18 x 2 + 0 (only in this stage the remainder becomes 0)  ∴ The HCF should be the last divisor, i.e., 18. |
| Convert fractions into decimals |

|  |
| --- |
| **Find the greatest number, which divides 285 and 1249, leaving remainders 9 and 7, respectively?**  (2015) |
| 135 |
| 136 |
| 137 |
| 138 |
| d |
| By applying Euclid’s division lemma, we get  1242 = 276 x 4 + 138 |
| 285 – 9 = 276 and 1249 -7 = 1242 can divide them exactly.  So, if the HCF of 276 and 1242 is found, then that will be the required number.  Now, by applying Euclid’s division lemma, we get  1242 = 276 x 4 + 138  276 = 138 x 2 + 0. (The remainder becomes 0 here)  So, the HCF = 138  ∴ The required number is 138. |
| Convert fractions into decimals |